ENSAE Paris - IP Paris
Academic year 2024-2025
First exam session (2A S1)

Final exam of the course "Macroeconomics 1"

Duration : 2 hours. No document allowed, no calculator allowed.
The grading scale, which may be modified, is indicated only for general-guidance purposes.

1 Lecture questions (6 points)

Answer very briefly the following questions (one or two sentences are enough for each question, and no equation is necessary).

Question 1 Indicate two important limitations of the Solow-Swan model.

Question 2 What does the economy interact with in the DICE model ? Or, in other words, what does the letter "C" in the acronym "DICE" stand for ?

Question 3 How do the private returns and the social returns of capital differ in the model of Romer (1986) ?

Question 4 In the model of Romer (1990), why can't the socially optimal equilibrium be implemented by subsidizing Research and Development ?

Question 5 Are government expenditures a stock or a flow ? And public debt ?

Question 6 Why is Ricardian equivalence not satisfied in the overlapping-generations model ?

2 Problem : automation/robotization in the Cass-Koopmans-Ramsey model (14 points)

The goal of this problem is to study the positive and normative consequences of a change in the production function in the Cass-Koopmans-Ramsey model. **One can answer each question without having answered the previous questions**, simply by using the results given in these previous questions.

We start from the Cass-Koopmans-Ramsey model considered in the lectures and the tutorials, in the particular case of a constant elasticity of intertemporal substitution, equal to $1/\theta$, where $\theta > 0$ and $\theta \neq 1$. We use exactly the same notations as in the lectures and the tutorials : r_t denotes the real interest rate, w_t the real wage, b_t the per-capita real amount of assets, L_t the population and aggregate labor supply, C_t aggregate consumption, c_t per-capita consumption, $\gamma_t \equiv C_t/(A_t L_t)$ consumption per effective-labor unit, K_t the aggregate capital stock, $\kappa_t \equiv K_t/(A_t L_t)$ the capital stock per effective-labor unit, and Y_t aggregate output at time t ; $\rho > 0$ denotes the rate of time preference, $n \geq 0$ the population-growth rate (i.e. the growth rate of population L_t), $g \geq 0$ the rate of technological progress (i.e. the growth rate of labor effectiveness A_t), and $\delta > 0$ the capital-depreciation rate. As in the lectures and the tutorials, we focus on parameter values such that $\rho - n > (1-\theta)g$. The only difference with the Cass-Koopmans-Ramsey model considered in the lectures and the tutorials is that we consider here the following production function :

$$Y_{i,t} = F(K_{i,t}, A_t N_{i,t}) \equiv Z K_{i,t} + K_{i,t}^{\alpha} \left(A_t N_{i,t}\right)^{1-\alpha},$$

where $Z \geq 0$, $0 < \alpha < 1$, and $Y_{i,t}$, $K_{i,t}$, $N_{i,t}$ are respectively the output, the capital stock, the labor demand of firm i at time t. We use the notation $f(x) \equiv F(x,1) = Zx + x^{\alpha}$.

2.1 Case $Z = 0$

We first consider the case $Z = 0$. In this case, the model is exactly the same as the one studied in the lectures and the tutorials, in the particular case of a Cobb-Douglas production function ($Y_{i,t} = K_{i,t}^{\alpha} \left(A_t N_{i,t}\right)^{1-\alpha}$).

Question 7 It is reminded that, in this model, the representative household's optimization problem is the following one : for some given $(r_t, w_t)_{t\geq 0}$ and b_0,

$$\max_{(c_t)_{t\geq 0}, (b_t)_{t>0}} \int_0^{+\infty} e^{-(\rho-n)t} \left(\frac{c_t^{1-\theta} - 1}{1-\theta}\right) dt$$

subject to

$$\forall t \geq 0,\ c_t \geq 0,$$

$$\forall t \geq 0,\ \dot{b}_t = (r_t - n) b_t + w_t - c_t,$$

$$\lim_{t\to+\infty} \left[b_t e^{-\int_0^t (r_\tau - n) d\tau}\right] \geq 0.$$

Interpret very briefly, in economic terms, the last two constraints (one or two sentences are enough for each constraint). Write the Hamiltonian of the representative household's problem, and then the first-order condition on the control variable and the costate equation. Deduce the Euler equation $\dot{c}_t/c_t = (r_t - \rho)/\theta$.

Question 8 Explain very briefly why we have $r_t = f'(\kappa_t) - \delta$ in equilibrium. Deduce the following differential equation :

$$\frac{\dot{\gamma}_t}{\gamma_t} = \frac{1}{\theta}\left[\frac{\alpha}{\kappa_t^{1-\alpha}} - (\delta + \rho + \theta g)\right].$$

Question 9 Write the goods-market-clearing condition in aggregate terms. Deduce the following differential equation :

$$\dot{\kappa}_t = \kappa_t^{\alpha} - \gamma_t - (n + g + \delta)\,\kappa_t.$$

Question 10 Define the steady state and show that κ_t and γ_t are constant over time at the steady state. Show that the saving rate $s_t \equiv (Y_t - C_t)/Y_t$ is also constant over time at the steady state, equal to $s^* \equiv \alpha(n + g + \delta)/(\delta + \rho + \theta g)$. Briefly interpret, in economic terms, the fact that s^* is strictly decreasing in ρ and θ.

2.2 Case $Z > 0$

In this section, we consider the case $Z > 0$.

Question 11 Of the five properties of the Cass-Koopmans-Ramsey model's production function (considered in the lectures and the tutorials), which ones does the production function considered here not satisfy ? Can we justify such a production function by invoking automation and robotization processes ?

Question 12 Write firms' optimization problem, get the first-order condition with respect to $K_{i,t}$, deduce that $K_{i,t}/N_{i,t}$ does not depend on i and is equal to K_t/L_t, and that $Y_t = F(K_t, A_t L_t)$.

Question 13 Explain very briefly how the answers to Questions 7, 8 and 9 are modified (if they are), and deduce the following two differential equations :

$$\frac{\dot{\gamma}_t}{\gamma_t} = \frac{1}{\theta}\left[\frac{\alpha}{\kappa_t^{1-\alpha}} - (\delta + \rho + \theta g - Z)\right],$$

$$\dot{\kappa}_t = \kappa_t^{\alpha} - \gamma_t - \left[(n + g + \delta) - Z\right]\kappa_t.$$

Question 14 Show that there exists a steady state if and only if

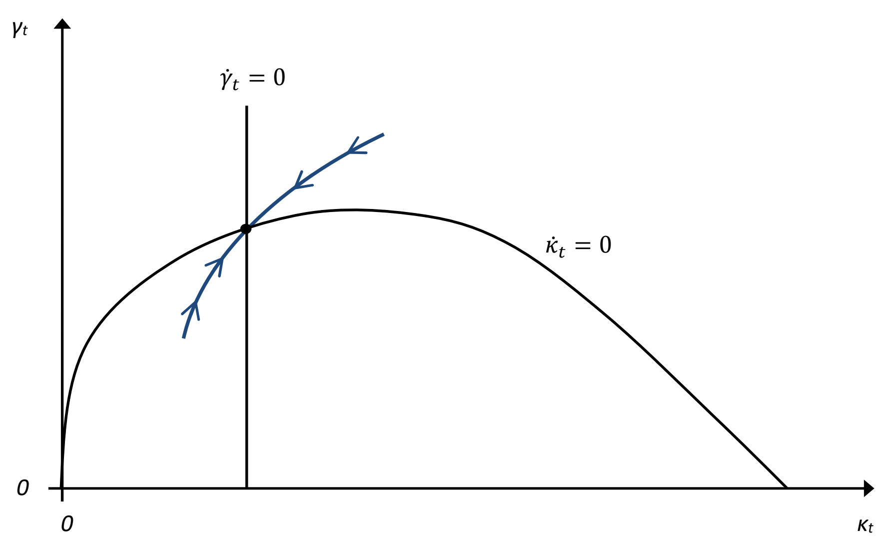
$$Z < \delta + \rho + \theta g.$$

Interpret this condition in economic terms. Assuming that this condition is satisfied, show that the saving rate s_t is constant over time at the steady state and determine its value as a function of the model's parameters. Does this value depend on Z, and how does it if it does ? Interpret in economic terms.

2.3 Case $0 < Z < n + g + \delta$

In this section, we consider the case $0 < Z < n + g + \delta$.

Question 15 Show that this case is a particular case of the case $0 < Z < \delta + \rho + \theta g$ (considered in the previous question). Briefly explain why the equations $\dot{\kappa}_t = 0$ and $\dot{\gamma}_t = 0$ correspond to a bell-shaped curve and a vertical straight line in the quadrant $(\kappa_t > 0, \gamma_t > 0)$ of the plane (κ_t, γ_t), as in the Cass-Koopmans-Ramsey model considered in the lectures and the tutorials, and as represented in the figure below. Show that the top of the bell-shaped curve is necessarily located on the right of the vertical straight line. What can we deduce in terms of the possibility of dynamic inefficiency ? Briefly explain.



Question 16 We admit that, in the absence of shocks and surprises, the unique equilibrium path is the saddle path, as in the Cass-Koopmans-Ramsey model considered in the lectures and the tutorials, and as represented in the figure above.[1] We assume that the economy is at the steady state until time T (excluded). At time T, unexpectedly, the parameter Z takes a new value, higher than its previous value, and remains at this new value thereafter. Draw the path followed by the economy in the plane (κ_t, γ_t). Explain and interpret. Is this path qualitatively different from the one obtained following a permanent reduction in parameter δ, and, if it is, how is it different ?

2.4 Case $Z > n + g + \delta$

In this section, we consider the case $Z > n + g + \delta$.

Question 17 In the sub-case $n+g+\delta < Z < \delta+\rho+\theta g$, draw, in the plane (κ_t, γ_t), the form that the equations $\dot{\kappa}_t = 0$ and $\dot{\gamma}_t = 0$ take. Can we have dynamic inefficiency in this case, and why ?

Question 18 In the sub-case $Z > \delta+\rho+\theta g$, show that $\gamma_t \to +\infty$ as $t \to +\infty$. Assuming that κ_t varies monotonously over time (i.e. either always decreasing, or always increasing, or always constant), show that $\kappa_t \to +\infty$ as $t \to +\infty$. Is the model an endogenous-growth model then ? Justify and interpret.

1. The proof of this result is essentially the same as the one seen in the lectures.