

Monetary Economics

Chapter 5: The Small-Open-Economy Extension

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# Goal of the chapter

- This chapter extends the basic NK model to an **open-economy setup** (nesting the closed economy as a special case).
- It introduces explicitly **key open-economy variables and concepts** such as
  - the exchange rate,
  - the terms of trade,
  - exports and imports,
  - international financial markets.
- It derives some important **positive and normative implications** of the openness of the economy for MP.

# Galí and Monacelli's (2005) model I

- We consider Galí and Monacelli's (2005) model, which is a model
  - of a **small open economy**, not affecting the rest of the world,
  - with **no international-trade cost**, so that the law of one price holds,
  - with **complete international financial markets**, allowing for consumption-risk sharing across countries.
  
- For simplicity, this model abstracts from
  - non-tradable goods,
  - nominal-wage stickiness,
  - cost-push shocks.

## Galí and Monacelli's (2005) model II

- In this model, the world economy is made of a **continuum** of infinitesimally small open economies represented by the unit interval  $[0, 1]$ .
- All these economies have the same **preferences, technology, and market structure**.
- The only shocks considered are **technology shocks**, which are imperfectly correlated across national economies.
- We consider a given small open economy, called the “**domestic economy**,” and we use the following notations:
  - variables without an  $i$  subscript refer to the domestic economy,
  - variables with an  $i$  subscript refer to the foreign economy  $i \in [0, 1]$ ,
  - variables with an asterisk superscript ( $*$ ) refer to the world economy.

# Main results of the chapter

- 1 There are two key equilibrium conditions, a **Phillips curve** and an **IS equation**, which are similar to their closed-economy counterparts.
- 2 There are **three sources of inefficiency**:
  - monopolistic competition,
  - price stickiness,
  - a terms-of-trade externality.
- 3 In a specific case, MP should have **two objectives**: stabilizing the output gap and “domestic inflation” (i.e. inflation in the price index for domestically produced goods).
- 4 In that specific case, **optimal MP** fully stabilizes domestic inflation.

# Outline of the chapter

- 1 Introduction
- 2 Households I
- 3 Households II
- 4 Firms
- 5 Equilibrium
- 6 Distortions
- 7 Loss function

# Utility function

- The representative household (RH) of the domestic economy maximizes

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t U [C_t, N_t] \right\},$$

where  $U$  is the instantaneous utility function, identical to Chapter 1's,  $N_t$  is work hours, and  $C_t$  is a **composite consumption index** defined by

$$C_t \equiv \left[ (1 - \alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where

- $C_{H,t}$  is an index of dom. consumption of dom. goods ( $H$  for *Home*),
- $C_{F,t}$  is an index of dom. consumption of foreign goods ( $F$  for *Foreign*),
- $\alpha \in [0, 1]$  is a measure of **openness** (the case  $\alpha = 0$  makes the model coincide with the closed-economy model studied in Chapters 1 to 3),
- $1 - \alpha$  is a measure of the degree of **home bias** in consumption,
- $\eta > 0$  is the dom. elasticity of subst. between dom. and foreign goods.

# Consumption indexes I

- The index of **domestic consumption of domestic goods** is defined as

$$C_{H,t} \equiv \left[ \int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where

- $C_{H,t}(j)$  denotes domestic consumption of domestic good  $j$ ,
  - $\varepsilon > 1$  is the elasticity of substitution between domestic goods.
- The index of **domestic consumption of foreign goods** is defined as

$$C_{F,t} \equiv \left[ \int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di \right]^{\frac{\gamma}{\gamma-1}},$$

where

- $C_{i,t}$  is an index of dom. consumption of goods produced in country  $i$ ,
- $\gamma > 1$  is the elasticity of substitution between goods produced in different countries.



# Consumption indexes II

- The fact that the definition of  $C_{F,t}$  involves an integral over the continuum  $[0, 1]$ , which includes the domestic economy, does not matter since the latter has a zero measure.
- The index of **domestic consumption of goods produced in country  $i$**  is defined in the same way as  $C_{H,t}$ :

$$C_{i,t} \equiv \left[ \int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where

- $C_{i,t}(j)$  denotes domestic consumption of good  $j$  produced in country  $i$ ,
- $\varepsilon$  is also the elasticity of subst. between goods produced in country  $i$ .

# Budget constraints

- RH faces the sequence of **budget constraints**

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + \mathbb{E}_t \{ Q_{t,t+1} D_{t+1} \} \leq D_t + W_t N_t + T_t$$

for  $t \geq 0$ , where

- $W_t$  is the nominal wage at date  $t$ ,
- $T_t$  is lump-sum transfers (or minus lump-sum taxes) at date  $t$ ,
- $P_{H,t}(j)$  is the price of domestic good  $j$  at date  $t$ ,
- $P_{i,t}(j)$  is the price of good  $j$  imported from country  $i$  at date  $t$ ,
- $D_{t+1}$  is the (random) nominal payoff at date  $t+1$  of the portfolio of securities bought by RH at date  $t$ ,

all of them expressed in units of domestic currency, and

- $Q_{t,t+1}$  is the stochastic discount factor for one-period-ahead nominal payoffs relevant to RH at date  $t$ .

# Distribution of consumption across goods I

- The optimized distribution of consumption across goods is characterized by five **demand schedules**, the first three of which are

$$C_{H,t}(j) = \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} C_{H,t}, \quad C_{i,t}(j) = \left[ \frac{P_{i,t}(j)}{P_{i,t}} \right]^{-\varepsilon} C_{i,t},$$
$$C_{i,t} = \left[ \frac{P_{i,t}}{P_{F,t}} \right]^{-\gamma} C_{F,t},$$

for all  $(i, j) \in [0, 1]^2$  and  $t \geq 0$ , where, at each date  $t \geq 0$ ,

- $P_{H,t} \equiv \left[ \int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$  is an index of prices of domestic goods,
- $P_{i,t} \equiv \left[ \int_0^1 P_{i,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$  is an index of prices of country  $i$ 's goods,
- $P_{F,t} \equiv \left[ \int_0^1 P_{i,t}^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}$  is an index of prices of imported goods,

all of them expressed in units of domestic currency.

# Distribution of consumption across goods II

- The last two **demand schedules** are

$$C_{H,t} = (1 - \alpha) \left[ \frac{P_{H,t}}{P_t} \right]^{-\eta} C_t,$$
$$C_{F,t} = \alpha \left[ \frac{P_{F,t}}{P_t} \right]^{-\eta} C_t,$$

for all  $t \geq 0$ , where, at each date  $t \geq 0$ ,

$$P_t \equiv \left[ (1 - \alpha) (P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

is the **consumer price index (CPI)**, expressed in units of domestic currency.

- When  $\eta \rightarrow 1$  or (as will be the case at the steady state)  $P_{H,t} = P_{F,t}$ , parameter  $\alpha$  corresponds to the share of domestic consumption allocated to imported goods, and represents therefore a **natural measure of openness**.

# Rewriting the budget constraints

- Combining the demand schedules with the definitions of price and consumption indexes, we get, in the same way as in Chapters 1 and 4,

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj = P_{H,t} C_{H,t},$$

$$\int_0^1 P_{i,t}(j) C_{i,t}(j) dj = P_{i,t} C_{i,t},$$

$$\int_0^1 P_{i,t} C_{i,t} di = P_{F,t} C_{F,t},$$

$$P_{H,t} C_{H,t} + P_{F,t} C_{F,t} = P_t C_t,$$

for  $i \in [0, 1]$  and  $t \geq 0$ , so that the date- $t$  **budget constraint** can be rewritten as

$$P_t C_t + \mathbb{E}_t \{ Q_{t,t+1} D_{t+1} \} \leq D_t + W_t N_t + T_t.$$

# Other intratemporal FOC of RH's optimization problem

- The other **intratemporal FOC** of RH's optimization problem is, as in Chapter 1,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}.$$

- As in Chapter 1, given that  $U(C_t, N_t) \equiv \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$ , it can be rewritten as

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\varphi,$$

or, in log-linearized form,

$$w_t - p_t = \sigma c_t + \varphi n_t.$$

# Intertemporal FOC of RH's optimization problem I

- Consider a given **Arrow security**, i.e. a one-period security that yields one unit of domestic currency if a specific state of nature is realized at date  $t + 1$  and nothing otherwise.
- The **intertemporal FOC** of RH's optimization problem can be written

$$\frac{V_{t,t+1} C_t^{-\sigma}}{P_t} = \frac{\tilde{\zeta}_{t,t+1} \beta C_{t+1}^{-\sigma}}{P_{t+1}},$$

where

- $V_{t,t+1}$  is the date- $t$  price (in domestic currency) of this Arrow security,
- $\tilde{\zeta}_{t,t+1}$  is the probability that this state of nature is realized at date  $t + 1$ , conditional on the state of nature at date  $t$ ,
- $C_{t+1}$  and  $P_{t+1}$  are here the values taken by the consumption index and the CPI at date  $t + 1$  when this state of nature is realized.

# Intertemporal FOC of RH's optimization problem II

- This FOC says that RH should be **indifferent** between purchasing one marginal unit of this Arrow security at date  $t$  or not:
  - the left-hand side is the utility **loss** resulting from the marginal decrease in  $C_t$  implied by this purchase,
  - the right-hand side is the utility **gain** resulting from the marg. increase in  $C_{t+1}$  in the corresponding state of nature implied by this purchase.
- The date- $t$  price of a portfolio yielding a random payoff  $D_{t+1}$  at date  $t + 1$  is

$$\sum_{\text{date-(}t+1\text{) states}} V_{t,t+1} D_{t+1} = \mathbb{E}_t \left\{ \frac{V_{t,t+1}}{\zeta_{t,t+1}} D_{t+1} \right\},$$

so that the **one-period stochastic discount factor** can be defined as

$$Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\zeta_{t,t+1}}.$$



# Intertemporal FOC of RH's optimization problem III

- Using this definition of  $Q_{t,t+1}$ , the previous FOC can be rewritten as

$$Q_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right)$$

for all possible states of nature at dates  $t$  and  $t + 1$ , which implies the same **Euler equation** as in Chapters 1 and 4:

$$Q_t = \beta \mathbb{E}_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right\},$$

where  $Q_t \equiv \mathbb{E}_t \{ Q_{t,t+1} \}$  is the date- $t$  price of a one-period bond paying off one unit of domestic currency at date  $t + 1$ , so that the first-order approximation of this Euler equation around the ZIRSS can again be written as

$$c_t = \mathbb{E}_t \{ c_{t+1} \} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \{ \pi_{t+1} \} - \bar{i}),$$

where  $i_t \equiv -\log Q_t$  is the short-term nominal interest rate,  $\bar{i} \equiv -\log \beta$  is the time-discount rate, and  $\pi_t \equiv p_t - p_{t-1}$  is the **CPI inflation rate**.

# Bilateral and effective terms of trade

- The **bilateral terms of trade** between the domestic economy and country  $i$  are defined as the price of country  $i$ 's goods in terms of home goods:

$$\mathcal{S}_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}.$$

- The **effective terms of trade** are defined and obtained as

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}} = \left( \int_0^1 \mathcal{S}_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}.$$

- Around a symmetric steady state with  $\mathcal{S}_{i,t} = 1$  for all  $i \in [0, 1]$ , they can be approximated, up to first order, by

$$s_t = \int_0^1 s_{i,t} di,$$

where  $s_t \equiv \log \mathcal{S}_t = p_{F,t} - p_{H,t}$ .

# Domestic and CPI inflation

- Around this symmetric steady state, the CPI definition can be approximated, up to first order, by

$$p_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t,$$

- Define **domestic inflation** as the rate of change in the index of domestic-goods prices:

$$\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}.$$

- Domestic inflation and **CPI inflation** are then linked by

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t,$$

i.e. the gap between the two measures of inflation is proportional to the change in the effective terms of trade, the coefficient of proportionality being given by the measure of openness  $\alpha$ .

# Law of one price and bilateral nominal exchange rate

- In the absence of international-trade cost, the **law of one price** holds:

$$P_{i,t}(j) = \varepsilon_{i,t} P_{i,t}^i(j)$$

for all  $(i, j) \in [0, 1]^2$ , where

- $\varepsilon_{i,t}$  is the **bilateral nominal exchange rate** with country  $i$  (i.e. the price of country  $i$ 's currency in terms of the domestic currency),
  - $P_{i,t}^i(j)$  is the price of country  $i$ 's good  $j$  expressed in its own currency.
- The law of one price and the definition of  $P_{i,t}$  together imply that

$$P_{i,t} = \varepsilon_{i,t} P_{i,t}^i$$

for all  $i \in [0, 1]$ , where  $P_{i,t}^i \equiv \left[ \int_0^1 P_{i,t}^i(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$  is country  $i$ 's domestic-price index expressed in its own currency.

# Effective nominal exchange rate and world-price index

- Using the previous result to replace  $P_{i,t}$  in the definition of  $P_{F,t}$ , we get, up to first order, around the symmetric steady state,

$$p_{F,t} = \int_0^1 (e_{i,t} + p_{i,t}^i) di = e_t + p_t^*,$$

where

- $e_{i,t} \equiv \log \mathcal{E}_{i,t}$ ,
  - $e_t \equiv \int_0^1 e_{i,t} di$  is the (log) **effective nominal exchange rate**,
  - $p_t^* \equiv \int_0^1 p_{i,t}^i di$  is the (log) **world-price index** (for the world as a whole, there is no distinction between the CPI and the domestic-price index).
- Therefore, the effective terms of trade can be written as

$$s_t = e_t + p_t^* - p_{H,t}.$$

# Bilateral and effective real exchange rates

- Define the **bilateral real exchange rate** with country  $i$  as the ratio of the two countries' CPIs, both expressed in terms of domestic currency:

$$Q_{i,t} \equiv \frac{\varepsilon_{i,t} P_t^i}{P_t}.$$

- Define the (log) **effective real exchange rate** as  $q_t \equiv \int_0^1 q_{i,t} di$ , where  $q_{i,t} \equiv \log Q_{i,t}$ .
- We then have, up to first order,

$$q_t = \int_0^1 \left( e_{i,t} + p_t^i - p_t \right) di = e_t + p_t^* - p_t = s_t + p_{H,t} - p_t = (1 - \alpha) s_t,$$

where the last “=” comes from the fact that  $\frac{P_t}{P_{H,t}} = \left[ (1 - \alpha) + \alpha S_t^{1-\eta} \right]^{\frac{1}{1-\eta}}$  can be approximated by  $p_t - p_{H,t} = \alpha s_t$  around a symmetric steady state.

# International risk sharing I

- Given that the Arrow securities are traded internationally, the **intertemporal FOC** of the optimization problem of any country  $i$ 's RH can be written as

$$\frac{V_{t,t+1} (C_t^i)^{-\sigma}}{\varepsilon_{i,t} P_t^i} = \frac{\tilde{\zeta}_{t,t+1} \beta (C_{t+1}^i)^{-\sigma}}{\varepsilon_{i,t+1} P_{t+1}^i}$$

for any Arrow security whose price ( $V_{t,t+1}$ ) and payoff (equal to 1) are expressed in domestic currency, where  $C_{t+1}^i$  and  $P_{t+1}^i$  are conditional on the state of nature corresponding to the Arrow security considered.

- In the same way as previously, this FOC is shown to imply

$$Q_{t,t+1} = \beta \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left( \frac{P_t^i}{P_{t+1}^i} \right) \left( \frac{\varepsilon_{i,t}}{\varepsilon_{i,t+1}} \right)$$

for all states of nature, all  $i \in [0, 1]$ , and all  $t \geq 0$ .

# International risk sharing II

- This equation and its domestic counterpart together imply

$$\frac{C_{t+1}}{C_t} = \left( \frac{C_{t+1}^i}{C_t^i} \right) \left( \frac{Q_{i,t+1}}{Q_{i,t}} \right)^{\frac{1}{\sigma}}$$

for all states of nature, all  $i \in [0, 1]$ , and all  $t \geq 0$ , which in turn implies

$$C_t = \vartheta_i C_t^i Q_{i,t}^{\frac{1}{\sigma}}$$

for all states of nature, all  $i \in [0, 1]$ , and all  $t \geq 0$ , where  $\vartheta_i$  is a constant depending on initial net foreign asset positions.

- We assume zero initial net foreign asset positions, so that  $\vartheta_i = 1$  for all  $i \in [0, 1]$  and the previous condition becomes, in aggregate log terms,

$$c_t = c_t^* + \frac{1}{\sigma} q_t \simeq c_t^* + \left( \frac{1 - \alpha}{\sigma} \right) s_t,$$

where  $c_t^* \equiv \int_0^1 c_t^i di$  denotes the (log) **world-consumption index**.



# Uncovered interest-rate parity

- The domestic-currency price of a riskless bond denominated in country  $i$ 's currency is

$$\varepsilon_{i,t} Q_t^i = \mathbb{E}_t \{ \varepsilon_{i,t+1} Q_{t,t+1} \},$$

where  $Q_t^i$  is the price of the bond in country  $i$ 's currency.

- This pricing equation, combined with the domestic-bond-pricing equation  $Q_t = \mathbb{E}_t \{ Q_{t,t+1} \}$  and the definition  $i_t^i \equiv -\log(Q_t^i)$ , implies

$$\mathbb{E}_t \left\{ Q_{t,t+1} \left[ \exp(i_t) - \frac{\varepsilon_{i,t+1}}{\varepsilon_{i,t}} \exp(i_t^i) \right] \right\} = 0.$$

- The latter condition, approximated around the steady state and aggregated over  $i \in [0, 1]$ , gives the **uncovered interest-rate parity**

$$i_t = i_t^* + \mathbb{E}_t \{ \Delta e_{t+1} \}.$$

# Technology

- In this chapter, for simplicity, we restrict the analysis to a **linear technology**:

$$Y_t(j) = A_t N_t(j),$$

where  $j \in [0, 1]$  indexes the continuum of firms.

- Therefore, the **real marginal cost** (expressed in domestic goods) is common across domestic firms and given by

$$mc_t = -\nu + w_t - p_{H,t} - a_t,$$

where  $\nu \equiv -\log(1 - \tau)$ , with  $\tau$  being the employment subsidy.

# Price setting

- As in Chapter 1, we assume that, at each date,
  - only a fraction  $1 - \theta$  of firms, drawn randomly from the population, are allowed to reset their price, where  $0 \leq \theta \leq 1$ ,
  - an individual firm's probability of being allowed to reset its price is independent of the time elapsed since it last reset its price.
- As shown in Chapter 1, the newly reset (log) domestic price at date  $t$ , noted  $\bar{p}_{H,t}$ , can be approximated as

$$\bar{p}_{H,t} = \mu + (1 - \beta\theta) \sum_{k=0}^{+\infty} (\beta\theta)^k \mathbb{E}_t \{ mc_{t+k} + p_{H,t+k} \},$$

where  $\mu \equiv \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$  is the (log) gross steady-state markup, or, equivalently, the (log) gross flexible-price markup.

# Goods-market-clearing conditions I

- The **domestic-goods-market-clearing conditions** are

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^i(j) di$$

for all  $j \in [0, 1]$  and  $t \geq 0$ , where  $C_{H,t}^i(j)$  denotes country  $i$ 's demand for domestic good  $j$ .

- Using the domestic demand schedules and the assumption of symmetric preferences across countries, we get

$$C_{H,t}^i(j) = \alpha \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} \left( \frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i} \right)^{-\gamma} \left( \frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i.$$

# Goods-market-clearing conditions II

- Therefore, the goods-market-clearing conditions can be rewritten as

$$Y_t(j) = \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} \left[ (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left( \frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i} \right)^{-\gamma} \left( \frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i di \right].$$

- Plugging this expression for  $Y_t(j)$  into  $Y_t \equiv \left[ \int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$  yields

$$\begin{aligned} Y_t &= (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left( \frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i} \right)^{-\gamma} \left( \frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i di \\ &= \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} \left[ (1 - \alpha) C_t + \alpha \int_0^1 \left( \frac{\varepsilon_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{\gamma-\eta} Q_{i,t}^\eta C_t^i di \right]. \end{aligned}$$

# Goods-market-clearing conditions III

- Using  $C_t = C_t^i Q_{i,t}^{\frac{1}{\sigma}}$ , we can rewrite the previous condition as

$$Y_t = \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \left[ (1 - \alpha) + \alpha \int_0^1 \left( S_t^i S_{i,t} \right)^{\gamma - \eta} Q_{i,t}^{\eta - \frac{1}{\sigma}} di \right],$$

where

- $S_t^i \equiv \frac{\varepsilon_{i,t} P_{F,t}^i}{P_{i,t}}$  is the effective terms of trade of country  $i$ ,
  - $S_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}$  is the bilateral terms of trade with country  $i$ .
- Using  $\int_0^1 s_t^i di = 0$ , we can approximate this condition around the symmetric steady state as

$$y_t = c_t + \alpha \gamma s_t + \alpha \left( \eta - \frac{1}{\sigma} \right) q_t = c_t + \frac{\alpha \omega}{\sigma} s_t,$$

where  $\omega \equiv \sigma \gamma + (1 - \alpha)(\sigma \eta - 1)$ .

# Goods-market-clearing conditions IV

- A similar condition holds for any country  $i \in [0, 1]$ :

$$y_t^i = c_t^i + \frac{\alpha\omega}{\sigma} s_t^i.$$

- By aggregating over countries  $i \in [0, 1]$  and using again  $\int_0^1 s_t^i di = 0$ , we get the **world goods-market-clearing condition**

$$y_t^* \equiv \int_0^1 y_t^i di = \int_0^1 c_t^i di \equiv c_t^*,$$

where  $y_t^*$  and  $c_t^*$  are (log) indexes for world output and consumption.

- Using this condition,  $c_t = c_t^* + \left(\frac{1-\alpha}{\sigma}\right) s_t$ , and  $y_t = c_t + \frac{\alpha\omega}{\sigma} s_t$ , we get

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t,$$

where  $\sigma_\alpha \equiv \frac{\sigma}{1+\alpha(\omega-1)} > 0$ .

# Rewriting the Euler equation

- Using sequentially  $y_t = c_t + \frac{\alpha\omega}{\sigma}s_t$ ,  $\pi_t = \pi_{H,t} + \alpha\Delta s_t$ , and  $y_t = y_t^* + \frac{1}{\sigma_\alpha}s_t$ , we can rewrite the **Euler equation** as

$$\begin{aligned} y_t &= \mathbb{E}_t \{y_{t+1}\} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \{\pi_{t+1}\} - \bar{i}) - \frac{\alpha\omega}{\sigma} \mathbb{E}_t \{\Delta s_{t+1}\} \\ &= \mathbb{E}_t \{y_{t+1}\} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \{\pi_{H,t+1}\} - \bar{i}) - \frac{\alpha\Theta}{\sigma} \mathbb{E}_t \{\Delta s_{t+1}\} \\ &= \mathbb{E}_t \{y_{t+1}\} - \frac{1}{\sigma_\alpha} (i_t - \mathbb{E}_t \{\pi_{H,t+1}\} - \bar{i}) + \alpha\Theta \mathbb{E}_t \{\Delta y_{t+1}^*\}, \end{aligned}$$

where  $\Theta \equiv \omega - 1$ .

- Thus, when  $\Theta > 0$  (i.e. for relatively large values of  $\eta$  and  $\gamma$ ), an increase in the degree of openness ( $\alpha$ ) raises the sensitivity ( $\frac{1}{\sigma_\alpha}$ ) of domestic output to the domestic real interest rate ( $i_t - \mathbb{E}_t \{\pi_{H,t+1}\}$ ), given world output.
- It does so by amplifying the **real appreciation** (and the consequent switch of expenditures towards foreign goods) induced by a given interest-rate rise.



# Trade balance

- Let  $nx_t \equiv \frac{1}{Y} \left( Y_t - \frac{P_t}{P_{H,t}} C_t \right)$  denote **net exports** in terms of domestic output, expressed as a fraction of steady-state output  $Y$ .
- We get, at the first order,

$$nx_t = y_t - c_t - \alpha s_t.$$

- Together with  $y_t = c_t + \frac{\alpha\omega}{\sigma} s_t$ , this implies

$$nx_t = \alpha \left( \frac{\omega}{\sigma} - 1 \right) s_t.$$

- Therefore, the relationship between net exports and the terms of trade may be positive or negative, depending on the values of the structural parameters.

# Aggregate production function

- Using the **individual production function**  $Y_t(j) = A_t N_t(j)$ , we get

$$N_t \equiv \int_0^1 N_t(j) dj = \frac{1}{A_t} \int_0^1 Y_t(j) dj = \frac{Y_t}{A_t} \int_0^1 \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} dj.$$

- Lemma 1 (established in Chapter 2) implies that variations in  $d_t \equiv \int_0^1 \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} dj$  around the steady state are of second order.

- We therefore get, at the first order, the following **aggregate production function**:

$$y_t = a_t + n_t.$$

# Domestic inflation and marginal cost

- As in Chapter 1, the equation describing the dynamics of the domestic-goods-price index as a function of newly set domestic prices,

$$\pi_{H,t} = (1 - \theta)(\bar{p}_{H,t} - p_{H,t-1}),$$

can be combined with the FOC of firms' optimization problem to yield

$$\pi_{H,t} = \beta \mathbb{E}_t \{ \pi_{H,t+1} \} + \chi \widehat{m\hat{c}}_t,$$

where  $\chi \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$ .

- Thus, the relationship between domestic inflation and the domestic marginal cost does not depend on any open-economy parameter.

# Marginal cost I

- Using  $w_t - p_t = \sigma c_t + \varphi n_t$ ,  $p_t - p_{H,t} = \alpha s_t$ ,  $c_t = c_t^* + \left(\frac{1-\alpha}{\sigma}\right) s_t$ ,  $c_t^* = y_t^*$ , and  $y_t = a_t + n_t$ , we can express the **domestic marginal cost**  $mc_t$  as

$$\begin{aligned}
 mc_t &= -v + (w_t - p_{H,t}) - a_t \\
 &= -v + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\
 &= -v + \sigma c_t + \varphi n_t + \alpha s_t - a_t \\
 &= -v + \sigma y_t^* + \varphi y_t + s_t - (1 + \varphi) a_t.
 \end{aligned}$$

- So the domestic marginal cost  $mc_t$  depends **positively on domestic output**  $y_t$ , through its effect on employment  $n_t$  and, hence, the real wage  $w_t - p_t$  (because of convex labor disutility:  $\varphi > 0$ ).
- It depends **negatively on technology**  $a_t$ , through
  - its direct effect on labor productivity,
  - its effect on  $n_t$  and, hence,  $w_t - p_t$ , for a given  $y_t$ .

# Marginal cost II

- It depends **positively on world output**  $y_t^*$ , through its effect on domestic consumption  $c_t$  (via international risk sharing) and, hence, the real wage  $w_t - p_t$  (because of concave consumption utility:  $\sigma > 0$ ).
- Lastly, it depends **positively on the terms of trade**  $s_t$ , through
  - their effect on  $c_t$  and, hence,  $w_t - p_t$ , for a given  $y_t^*$ ,
  - their direct effect on  $w_t - p_{H,t}$  for a given  $w_t - p_t$ .

- Using  $y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t$ , we can rewrite  $mc_t$  as

$$mc_t = -v + (\sigma_\alpha + \varphi) y_t + (\sigma - \sigma_\alpha) y_t^* - (1 + \varphi) a_t.$$

- So domestic output  $y_t$  affects the domestic marginal cost  $mc_t$  through
  - its effect on employment (captured by  $\varphi$ ),
  - its effect on the terms of trade (captured by  $\sigma_\alpha$ ).

# Marginal cost III

- World output  $y_t^*$  affects the domestic marginal cost  $mc_t$  through
  - its effect on consumption (captured by  $\sigma$ ),
  - its effect on the terms of trade (captured by  $\sigma_\alpha$ ).
  
- When  $\Theta > 0$  (i.e. for relatively high substitutability between goods produced in different countries), we have  $\sigma > \sigma_\alpha$ , so that the domestic marginal cost  $mc_t$  depends **positively on world output**  $y_t^*$ .
  
- The reason is that the size of the real appreciation needed to absorb the change in relative supplies is then relatively small.
  
- When  $\Theta > 0$ , an increase in openness  $\alpha$ 
  - decreases the sensitivity of  $mc_t$  (and hence  $\pi_{H,t}$ ) to  $y_t$ ,
  - increases the sensitivity of  $mc_t$  (and hence  $\pi_{H,t}$ ) to  $y_t^*$ ,
 by reducing the size of the required adjustment in the terms of trade.

## Natural level of output and output gap

- Define the **natural level of output**  $y_t^n$  as the level of domestic output that would prevail if prices were flexible in the domestic economy and sticky elsewhere.
- Since the firms' FOC implies that  $mc_t = -\mu$  under flexible prices, we get

$$y_t^n = \Gamma_0 + \Gamma_a a_t + \Gamma_* y_t^*,$$

where  $\Gamma_0 \equiv \frac{\nu - \mu}{\sigma_\alpha + \varphi}$ ,  $\Gamma_a \equiv \frac{1 + \varphi}{\sigma_\alpha + \varphi}$ , and  $\Gamma_* \equiv -\frac{\alpha \Theta \sigma_\alpha}{\sigma_\alpha + \varphi}$  ( $\leq 0$  depending on the relative importance of the terms-of-trade effect discussed above).

- Using the last expressions for  $mc_t$  and  $y_t^n$ , we get

$$\widehat{mc}_t = (\sigma_\alpha + \varphi) \tilde{y}_t,$$

where  $\tilde{y}_t \equiv y_t - y_t^n$  is the **output gap**.

# Phillips curve

- Using the latter expression to replace  $\widehat{mc}_t$  in the firms's FOC, we get the following **Phillips curve**:

$$\pi_{H,t} = \beta \mathbb{E}_t \{ \pi_{H,t+1} \} + \kappa_\alpha \tilde{y}_t,$$

where  $\kappa_\alpha \equiv (\sigma_\alpha + \varphi) \chi$ .

- This small-open-economy Phillips curve is **isomorphic** to its closed-economy counterpart.
- The **main difference** is that the degree of openness  $\alpha$  affects the slope  $\kappa_\alpha$  of the small-open-economy Phillips curve.
- More precisely, when  $\Theta > 0$ , an increase in  $\alpha$  decreases  $\kappa_\alpha$ , by reducing the real depreciation induced by an increase in domestic output and, hence, the effect of the latter on marginal cost and inflation.



# IS equation

- Using the expression for  $y_t^n$  to rewrite the Euler equation, we get the following **IS equation**:

$$\tilde{y}_t = \mathbb{E}_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma_\alpha} (i_t - \mathbb{E}_t \{ \pi_{H,t+1} \} - r_t^n),$$

where  $r_t^n \equiv \bar{i} - \sigma_\alpha \Gamma_\alpha (1 - \rho_a) a_t + \frac{\alpha \ominus \sigma_\alpha \varphi}{\sigma_\alpha + \varphi} \mathbb{E}_t \{ \Delta y_{t+1}^* \}$  is the domestic **natural rate of interest**.

- This small-open-economy IS equation is **isomorphic** to its closed-economy counterpart.
- The **main differences** are that, in the small-open-economy IS equation,
  - the degree of openness  $\alpha$  influences the sensitivity  $\frac{1}{\sigma_\alpha}$  of the output gap to interest-rate changes,
  - the natural rate of interest  $r_t^n$  depends on expected world-output growth  $\mathbb{E}_t \{ \Delta y_{t+1}^* \}$ , in addition to domestic productivity  $a_t$ .

# Taylor principle

- Given  $(a_t, i_t)_{t \in \mathbb{N}}$ ,  $(\tilde{y}_t, \pi_{H,t})_{t \in \mathbb{N}}$  is determined by
  - the IS equation  $\tilde{y}_t = \mathbb{E}_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma_\alpha} (i_t - \mathbb{E}_t \{ \pi_{H,t+1} \} - r_t^n)$ ,
  - the Phillips curve  $\pi_{H,t} = \beta \mathbb{E}_t \{ \pi_{H,t+1} \} + \kappa_\alpha \tilde{y}_t$ ,

for  $t \in \mathbb{N}$ .

- Given the isomorphism between the closed- and small-open-economy Phillips curve and IS equation, we get the same **determinacy conditions** as in Chapter 3 for the same parametric families of rules, except that
  - $\sigma$  and  $\kappa$  should be replaced by  $\sigma_\alpha$  and  $\kappa_\alpha$  in the conditions,
  - $\pi$  and  $x$  should be replaced by  $\pi_H$  and  $\tilde{y}$  in the rules.
- Therefore, we get the same **Taylor principle** as in Chapter 3: in the long term, the (nominal) interest rate should rise by more than the increase in the domestic inflation rate in order to ensure determinacy.

# A special case

- In the rest of the chapter (devoted to normative issues), we focus on the **special case** in which  $\sigma = \eta = \gamma = 1$ .
- In this special case, the following equilibrium conditions, previously obtained as first-order approximations, hold **exactly**:

$$s_t = \int_0^1 s_{i,t} di, \quad p_t = p_{H,t} + \alpha s_t, \quad \pi_t = \pi_{H,t} + \alpha \Delta s_t,$$

$$q_t = (1 - \alpha) s_t, \quad c_t = c_t^* + \left( \frac{1 - \alpha}{\sigma} \right) s_t = c_t^* + (1 - \alpha) s_t,$$

$$y_t = c_t + \frac{\alpha \omega}{\sigma} s_t = c_t + \alpha s_t, \quad nx_t = 0.$$

- Moreover, we have  $\omega = 1$ ,  $\sigma_\alpha = \sigma$ ,  $\Theta = 0$ ,  $\Gamma_* = 0$ , and  $\kappa_\alpha = \kappa$ , so that the small-open-economy IS equation and Phillips curve are exactly **identical** to their closed-economy counterparts.

# Social-planner allocation I

- Consider a **benevolent social planner**, seeking to maximize the welfare of the domestic economy's RH, subject to
  - the technology constraint,
  - the same resource constraints as those faced by the domestic economy vis-à-vis the rest of the world, given the complete-markets assumption.
- Given the absence of state variable (such as the capital stock), its optimization problem is **static**: at each date  $t$ , taking  $C_t^*$  as given,

$$\underset{C_t, N_t}{\text{Max}} U(C_t, N_t)$$

subject to

- the tech. constraint  $Y_t = A_t N_t$  (output being the same across goods),
- the international-risk-sharing condition  $C_t = C_t^* S_t^{1-\alpha}$ ,
- the goods-market-clearing condition  $Y_t = C_t S_t^\alpha$ .

# Social-planner allocation II

- These three constraints, together with the world goods-market-clearing condition  $C_t^* = Y_t^*$ , can be summarized by

$$C_t = A_t^{1-\alpha} (Y_t^*)^\alpha N_t^{1-\alpha}.$$

- The **optimality condition** equalizes the MRS between consumption and work to the corresponding marginal rate of transformation:

$$-\frac{U_{n,t}}{U_{c,t}} = (1 - \alpha) \frac{C_t}{N_t},$$

which implies

$$N_t = (1 - \alpha)^{\frac{1}{1+\varphi}} \quad \text{and} \quad C_t = (1 - \alpha)^{\frac{1-\alpha}{1+\varphi}} A_t^{1-\alpha} (Y_t^*)^\alpha.$$

- Thus, at the **social-planner allocation**,
  - employment  $N_t$  is constant over time,
  - cons.  $C_t$  fluctuates in response to technology  $A_t$  and world output  $Y_t^*$ .

# Distortions

- The model is characterized by **three distortions**:
  - ① monopolistic competition in the goods market,
  - ② price stickiness,
  - ③ a terms-of-trade externality between countries.
- The **first two distortions** are the same as in Chapters 2 and 4.
- As noted by Corsetti and Pesenti (2001) and Benigno and Benigno (2003), the **third distortion** comes from the CB's ability to influence the terms of trade in a way beneficial to domestic consumers, due to
  - the imperfect substitutability between domestic and foreign goods,
  - price stickiness, making MP not neutral.

# Condition for natural-allocation efficiency I

- Define the **natural allocation** as the equilibrium allocation when prices are flexible in the domestic economy and sticky elsewhere.
- Since  $\eta = 1$ , we have  $P_t = P_{H,t}^{1-\alpha} P_{F,t}^\alpha$  and hence  $\frac{P_t}{P_{H,t}} = \left(\frac{P_{F,t}}{P_{H,t}}\right)^\alpha = S_t^\alpha$ .
- Using this result, RH's intratemporal FOC  $-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$ , the goods-market-clearing condition  $Y_t = C_t S_t^\alpha$ , and the flexible-price aggregate production function  $Y_t = A_t N_t$ , we can characterize the natural allocation by

$$\begin{aligned} \frac{\varepsilon - 1}{\varepsilon} &= MC_t = \frac{(1 - \tau)W_t}{A_t P_{H,t}} = \frac{(1 - \tau)W_t S_t^\alpha}{A_t P_t} = -\frac{(1 - \tau)S_t^\alpha U_{n,t}}{A_t U_{c,t}} \\ &= -\frac{(1 - \tau)Y_t U_{n,t}}{A_t C_t U_{c,t}} = \frac{(1 - \tau)Y_t N_t^\varphi C_t}{A_t C_t} = (1 - \tau)N_t^{1+\varphi}. \end{aligned}$$

## Condition for natural-allocation efficiency II

- Therefore, the value

$$\tau = 1 - \frac{\varepsilon - 1}{(1 - \alpha)\varepsilon}$$

is such that the corresponding **natural allocation is efficient** (i.e. coincides with the social-planner allocation).

- This value depends not only on the elasticity of substitution between goods  $\varepsilon$ , as in Chapter 2, but also on the degree of openness  $\alpha$ .
- This is because openness creates a **terms-of-trade externality** between countries, which distorts the incentives of CB beyond monop. competition.
- I.e., an employment subsidy exactly offsetting the monopolistic-competition distortion would not make the natural allocation efficient, because CB would have an incentive to deviate from it in order to improve the terms of trade.



# MP and the (efficient) natural allocation I

- In the rest of the chapter, we assume that  $\tau = 1 - \frac{\varepsilon-1}{(1-\alpha)\varepsilon}$ , so that the natural allocation is efficient.
- As in Chapter 2, and as apparent from the IS equation and the Phillips curve, MP can then achieve the (efficient) natural allocation ( $\tilde{y}_t = 0$ ) by
  - making the interest rate track the natural rate of interest:  $i_t = r_t^n$ ,
  - **stabilizing domestic inflation** at a constant (zero) level:  $\pi_{H,t} = 0$ .
- As in Chapter 2, this is because the flexible-price allocation can be replicated when prices are sticky by making all firms satisfied with their existing prices, so that **the sticky-price constraint is not binding**.
- Because it perfectly stabilizes domestic inflation, this optimal MP is sometimes called “**(strict) domestic-inflation targeting**” (DIT).

# MP and the (efficient) natural allocation II

- Under DIT, in response to a positive tech. shock, for given world variables,
  - domestic output increases:  $y_t = \Gamma_0 + \Gamma_a a_t$  with  $\Gamma_a > 0$ ,
  - the terms of trade increase, i.e. deteriorate:  $s_t = \sigma(y_t - y_t^*)$ ,
  - the real exchange rate increases, i.e. depreciates:  $q_t = (1 - \alpha)s_t$ ,
  - the nom. exch. rate increases, i.e. depreciates:  $e_t = s_t - p_t^* + p_{H,t}$ ,
  - the CPI increases:  $p_t = p_{H,t} + \alpha s_t$ .
  
- Under DIT and constant world prices  $p_t^*$ , **the lower the correlation** between domestic natural output  $y_t^n$  (or, equivalently, domestic productivity  $a_t$ ) and world output  $y_t^*$ , **the higher the volatility** of
  - the terms of trade  $s_t$ ,
  - the real exchange rate  $q_t$ ,
  - the nominal exchange rate  $e_t$ ,
  - the CPI  $p_t$ .

# MP and the (efficient) natural allocation III

- Under DIT, for a given correlation between domestic natural output  $y_t^n$  and world output  $y_t^*$ , an increase in the degree of openness  $\alpha$ 
  - has no effect on the volatility of domestic output  $y_t$ ,
  - has no effect on the volatility of the terms of trade  $s_t$ ,
  - has no effect on the volatility of the nominal exchange rate  $e_t$ ,
  - decreases the volatility of the real exchange rate  $q_t$ ,
  - increases the volatility of the CPI  $p_t$ .
  
- Thus, optimal MP (i.e. DIT) may entail **large movements in the nominal exchange rate and CPI inflation**, especially for an economy that is very open and subject to largely idiosyncratic shocks.
  
- This is because optimal MP allows the nominal exchange rate and CPI inflation to adjust as needed in order to replicate the flexible-price response of the terms of trade, given that domestic prices are constant.

# Motivation for determining the welfare-loss function

- The **efficient allocation** is feasible only if CB observes  $a_t$  or some date- $t$  endogenous variables at each date  $t$  (so that  $i_t = r_t^n$  in equilibrium).
- When this condition is not met, MP cannot achieve the efficient allocation, so that **optimal feasible MP** cannot be derived from the efficient allocation.
- In that case, optimal feasible MP is obtained by minimizing the **welfare-loss function**, i.e. the second-order approximation of RH's utility function, subject to the structural equations and CB's observation-set constraint.
- This welfare-loss function tells us
  - the **objectives** that MP should have,
  - the **weight** that CB should put on each objective.

# Determination of the welfare-loss function I

- We now derive the second-order approximation of RH's **intertemporal utility** in the neighborhood of the symmetric steady state.
- The exact relationships  $c_t = c_t^* + (1 - \alpha) s_t$  and  $y_t = c_t + \alpha s_t$  imply that **instantaneous consumption utility** can be written exactly as

$$\log C_t = c_t = (1 - \alpha) y_t + \alpha c_t^* = (1 - \alpha) \hat{y}_t + t.i.p.,$$

where *t.i.p.* stands for “terms independent of policy.”

- As in Chapters 2 and 4, note that, for any variable  $Z_t$ , we have

$$\frac{Z_t - Z}{Z} \simeq \hat{z}_t + \frac{\hat{z}_t^2}{2},$$

where  $\hat{z}_t \equiv z_t - z$  is the log-deviation of  $Z_t$  from its steady-state value.

# Determination of the welfare-loss function II

- Therefore, **instantaneous labor disutility** can be approximated, up to second order, as

$$\begin{aligned} \frac{N_t^{1+\varphi}}{1+\varphi} &\simeq \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi} \left( \frac{N_t - N}{N} \right) + \frac{\varphi N^{1+\varphi}}{2} \left( \frac{N_t - N}{N} \right)^2 \\ &\simeq \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi} \left( \hat{n}_t + \frac{1+\varphi}{2} \hat{n}_t^2 \right). \end{aligned}$$

- Now, recall that

$$\hat{y}_t = \hat{n}_t + a_t - d_t,$$

where  $d_t \equiv \int_0^1 \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} dj$ .

# Determination of the welfare-loss function III

- Moreover, we know from Lemma 1 (established in Chapter 2) that, up to a second-order approximation,

$$d_t \simeq \frac{\varepsilon}{2} \text{var}_j \{p_{H,t}(j)\}.$$

- We can therefore rewrite **instant. labor disutility**, up to second order, as

$$\frac{N_t^{1+\varphi}}{1+\varphi} \simeq \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi} \left[ \hat{y}_t + \frac{\varepsilon}{2} \text{var}_j \{p_{H,t}(j)\} + \frac{1+\varphi}{2} (\hat{y}_t - a_t)^2 \right] + t.i.p.$$

- Our optimal-employment-subsidy assumption,  $\tau = 1 - \frac{\varepsilon-1}{(1-\alpha)\varepsilon}$ , implies

$$N^{1+\varphi} = 1 - \alpha.$$

# Determination of the welfare-loss function IV

- Therefore, a second-order approximation of **instantaneous utility** is

$$\begin{aligned} \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} &\simeq -(1-\alpha) \left[ \frac{\varepsilon}{2} \text{var}_j \{p_{H,t}(j)\} + \frac{1+\varphi}{2} (\hat{y}_t - a_t)^2 \right] + t.i.p. \\ &\simeq -(1-\alpha) \left[ \frac{\varepsilon}{2} \text{var}_j \{p_{H,t}(j)\} + \frac{1+\varphi}{2} (\tilde{y}_t)^2 \right] + t.i.p., \end{aligned}$$

where the second equality comes from the fact that, at the first order,

$$\hat{y}_t - a_t = y_t - y - a_t \simeq y_t - y_t^n = \tilde{y}_t.$$

- Finally, we know from Lemma 2 (stated in Chapter 2) that, up to a second-order approximation,

$$\sum_{t=0}^{+\infty} \beta^t \text{var}_j \{p_{H,t}(j)\} \simeq \frac{1}{\chi} \sum_{t=0}^{+\infty} \beta^t (\pi_{H,t})^2.$$



# Determination of the welfare-loss function $V$

- Therefore, we get that, up to second order,  $\mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t U_t \right\} \simeq$   

$$- \left( \frac{1-\alpha}{2} \right) \mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t \left[ \frac{\varepsilon}{\chi} (\pi_{H,t})^2 + (1+\varphi) (\tilde{y}_t)^2 \right] \right\} + t.i.p.$$

- Hence the **welfare-loss function**

$$L_0 \equiv \mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t \left[ (\pi_{H,t})^2 + \lambda (\tilde{y}_t)^2 \right] \right\},$$

where  $\lambda \equiv \frac{(1+\varphi)\chi}{\varepsilon}$ .

# Determination of the welfare-loss function VI

- This welfare-loss function is **identical** to its closed-economy counterpart, obtained in Chapter 2, for
  - no steady-state distortion,
  - no cost-push shocks,
  - constant returns to scale,
  - an elasticity of intertemporal substitution equal to one,with dom. inflation, not CPI inflation, being the relevant inflation variable.
  
- It can be **interpreted** in exactly the same way as in Chapter 2.