Monetary Economics

Chapter 1: The Basic New Keynesian Model

Olivier Loisel

ENSAE

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Objective of the chapter

- The chapter presents the **basic NK model** and derives its implications regarding the role of expectations in the transmission of monetary policy.
- The basic NK model corresponds to the standard RBC model with no capital and, in the goods market,
  - **monopolistic competition** (so firms are price-makers, not -takers),
  - **price stickiness**.
- It is a **cashless** model, in which money implicitly serves only as a unit of account.
- This model will be used in Chapters 2, 3, and 4.

Time and agents

- Time is discrete, indexed by t, and the horizon is infinite.
- There are **four kinds of agents**:
  - a large number of households,
  - a large number of firms,
  - a single monetary authority,
  - a single fiscal authority.
- All of them are infinitely lived.
- All households are identical, so that there is a representative household (RH).

Markets

- A continuum of **monopolistically** competitive **goods** markets:
  - demand from households,
  - supply from firms.
- A **perfectly** competitive **labor** market:
  - demand from firms,
  - supply from households.
- A **perfectly** competitive **one-period-bond** market:
  - demand from households,
  - supply from households.

Outline of the chapter

- Introduction
- Households' behavior
- Firms' behavior
- Equilibrium conditions
- Private sector's expectations

RH's intertemporal utility

- RH's **intertemporal utility** function at date 0 is

$$\mathcal{U}_0 \equiv \mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t U\left(C_t, N_t\right) \right\}, \text{ with } C_t \equiv \underbrace{\left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}}_{\text{constant-elasticity-of-substitution (CES) aggregator or Dixit-Stiglitz (1977) aggregator}},$$

where

- $\beta \in (0, 1)$ is the discount factor,
- C_t is the consumption index,
- $C_t(i)$ is the consumption of good i,
- $\varepsilon > 1$ is the elasticity of substitution between differentiated goods (with the limit case $\varepsilon \to +\infty$ corresponding to perfect competition),
- N_t is hours worked,
- $U_{c,t} > 0$, $U_{cc,t} < 0$, $U_{n,t} < 0$, $U_{nn,t} < 0$.

RH's optimization problem in words

- RH chooses how much
  - of each good to consume,
  - labor to supply,
  - bonds to hold,

  in order to maximize
  - her intertemporal utility function,

  subject to
  - her budget constraint,

  taking as given
  - the price of each good,
  - the wage,
  - the price of bonds

  (given the large number of households).

RH's optimization problem formalized

At date 0,

$$\underset{\substack{[C_t(i)]_{i\in[0,1],t\in\mathbb{N}},\\ (N_t)_{t\in\mathbb{N}},(B_t)_{t\in\mathbb{N}}}}{Max} \quad \mathbb{E}_0\left\{\sum_{t=0}^{+\infty}\beta^t U\left(C_t,N_t\right)\right\}$$

subject to

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{and}$$

$$\int_0^1 P_t(i)C_t(i)di + Q_tB_t \leq B_{t-1} + W_tN_t + T_t \quad \text{for} \quad t\in\mathbb{N},$$

taking as given

$$B_{-1},\ [P_t(i)]_{0\leq i\leq 1},\ Q_t,\ W_t, \text{ and } T_t \quad \text{for} \quad t\in\mathbb{N}.$$

Notations and resolution

- **Notations**:
  - $P_t(i)$ is the price of good i,
  - Q_t is the price of one-period nominal bonds (paying one unit of money at maturity),
  - B_t is the quantity of one-period nominal bonds held by RH,
  - W_t is the nominal wage,
  - T_t is a lump-sum transfer (if positive) or lump-sum tax (if negative).

- RH's optimization problem can be solved in **two steps**:
  1. for any given consumption index C_t, characterize RH's choice of the distribution of consumption across goods $[C_t(i)]_{0 \leq i \leq 1}$,
  2. characterize RH's choice of the consumption index C_t and the number of hours worked N_t.

Distribution of consumption across goods I

- **Dual optimization problem**:

$$\underset{[C_t(i)]_{0\leq i\leq 1}}{Max}\left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

subject to $\int_0^1 P_t(i)C_t(i)di = Z_t$.

- **Lagrangian**:

$$L=\left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}-\lambda\left[\int_0^1 P_t(i)C_t(i)di - Z_t\right].$$

- **First-order conditions** (FOCs): $C_t(i)^{\frac{-1}{\varepsilon}}C_t^{\frac{1}{\varepsilon}}=\lambda P_t(i)$ for all $i\in[0,1]$.

Distribution of consumption across goods II

- Using these FOCs to replace $C_t(i)$ in the definition of C_t gives $\lambda = P_t^{-1}$ where

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$$

is the **aggregate price index**.

- Replacing λ by P_t^{-1} in the FOCs gives the **demand schedule**

$$C_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon} C_t \text{ for all } i \in [0,1].$$

- So, if $P_t(i)$ is one percentage point higher than P_t, then $C_t(i)$ is (approximately) ε percentage points lower than C_t.

Consumption index and hours worked I

- Replacing $C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}$ by $C_t(i)P_t(i)C_t^{\frac{-1}{\varepsilon}}P_t^{-1}$ in the definition of C_t gives

$$\int_0^1 P_t(i)C_t(i)di = P_t C_t.$$

- Therefore, the second step of RH's optimization problem can be rewritten as

$$\underset{(C_t)_{t\in\mathbb{N}},(N_t)_{t\in\mathbb{N}},(B_t)_{t\in\mathbb{N}}}{Max} \mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t U\left(C_t, N_t\right) \right\}$$

subject to

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \quad \text{for} \quad t \in \mathbb{N},$$

taking as given

$$B_{-1},\ P_t,\ Q_t,\ W_t, \text{ and } T_t \quad \text{for} \quad t \in \mathbb{N}.$$

Consumption index and hours worked II

- **Lagrangian** at date 0:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t \left[U\left(C_t, N_t\right) - \lambda_t \left(P_t C_t + Q_t B_t - B_{t-1} - W_t N_t - T_t\right)\right] \right\}.$$

- **Lagrangian** at any date $t \in \mathbb{N}$:

$$\mathbb{E}_t \left\{ \sum_{k=0}^{+\infty} \beta^k \left[U\left(C_{t+k}, N_{t+k}\right) - \lambda_{t+k} \left(P_{t+k} C_{t+k} + Q_{t+k} B_{t+k} - B_{t+k-1} - W_{t+k} N_{t+k} - T_{t+k}\right)\right] \right\}.$$

- **FOCs**:

$$U_{c,t} = \lambda_t P_t,$$
$$U_{n,t} = -\lambda_t W_t,$$
$$\lambda_t Q_t = \beta \mathbb{E}_t \left\{ \lambda_{t+1} \right\}.$$

Consumption index and hours worked III

- These FOCs can be rewritten as

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \quad (\textbf{labor-consumption trade-off condition}),$$

$$Q_t = \beta \mathbb{E}_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \quad (\textbf{Euler equation}).$$

- **Interpretation**: at the optimal plan, it must be the case that
  - $U_{c,t} dC_t + U_{n,t} dN_t = 0$ for any pair (dC_t, dN_t) satisfying the budget constraint $P_t dC_t = W_t dN_t$,
  - $U_{c,t} dC_t + \beta \mathbb{E}_t \{ U_{c,t+1} dC_{t+1} \} = 0$ for any pair (dC_t, dC_{t+1}) satisfying the budget constraint $P_{t+1} dC_{t+1} = -\frac{P_t}{Q_t} dC_t$.

Consumption index and hours worked IV

- Consider **isoelastic** consumption-utility and labor-disutility functions:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where $\sigma > 0$ and $\varphi > 0$, with

  - σ the coefficient of relative risk aversion,
  - $1/\sigma$ the elasticity of intertemporal substitution,
  - $1/\varphi$ the Frisch elasticity of labor supply.

- The previous FOCs then become

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \quad \textbf{(labor-consumption trade-off condition)},$$

$$Q_t = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \quad \textbf{(Euler equation)}.$$

Consumption index and hours worked V

- The **labor-consumption trade-off condition** can be rewritten in log-linear form as

$$w_t - p_t = \sigma c_t + \varphi n_t,$$

where, for any variable Z_t, $z_t \equiv \log Z_t$.

- The log-linear approximation of the **Euler equation** (for small fluctuations) is

$$c_t = \mathbb{E}_t \{c_{t+1}\} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \{\pi_{t+1}\} - r),$$

where $i_t \equiv \log(1/Q_t) = \log[1 + (1 - Q_t)/Q_t] \approx (1 - Q_t)/Q_t$ is the **short-term nominal interest rate**, $r \equiv \log(1/\beta)$, and $\pi_t \equiv p_t - p_{t-1}$ is the **inflation rate**.

Production function and price rigidity

- There is a continuum of firms indexed by $i \in [0, 1]$, each firm producing a differentiated good.
- All firms use the same technology, represented by the **production function**

$$Y_t(i) = A_t N_t(i)^{1-\alpha},$$

where $\alpha \in [0, 1)$ and A_t is a stochastic exogenous factor (technology shock).

- **Prices are rigid** as in Calvo (1983): at each date, each firm can reset its price only with probability $1 - \theta$ (independent across time and firms), where $\theta \in [0, 1)$, so that
  - at each date, a measure $1 - \theta$ of firms reset their prices,
  - at each date, a measure θ of firms keep their prices unchanged,
  - the average duration of a price is $(1 - \theta)^{-1}$,
  - θ is a natural index of price stickiness.

Aggregate price level dynamics I

- At each date t, all firms resetting their price will choose the same price noted P_t^* because they face the same problem. So:

$$
\begin{aligned}
P_t &\equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \\
&= \left[\int_{\substack{\text{non-resetting} \\ \text{firms}}} P_t(i)^{1-\varepsilon} di + \int_{\substack{\text{resetting} \\ \text{firms}}} P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \\
&= \left[\int_{\substack{\text{non-resetting} \\ \text{firms}}} P_{t-1}(i)^{1-\varepsilon} di + \int_{\substack{\text{resetting} \\ \text{firms}}} \left(P_t^*\right)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \\
&= \left[\theta \int_0^1 P_{t-1}(i)^{1-\varepsilon} di + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \\
&= \left[\theta \left(P_{t-1}\right)^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.
\end{aligned}
$$

Aggregate price level dynamics II

- Dividing by P_{t-1} and defining $\Pi_t \equiv P_t / P_{t-1}$, we get

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

$$\text{and hence} \quad \Pi_t^{1-\varepsilon} - 1 = (1-\theta)\left[\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} - 1\right].$$

- For small fluctuations of Π_t around 1, we have

$$\Pi_t^{1-\varepsilon} - 1 \approx \log\left(1 + \Pi_t^{1-\varepsilon} - 1\right) = (1-\varepsilon)\,\pi_t,$$

$$\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} - 1 \approx \log\left[1 + \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} - 1\right] = (1-\varepsilon)\,(p_t^* - p_{t-1}),$$

$$\text{and therefore} \quad \pi_t = (1-\theta)(p_t^* - p_{t-1}).$$

Firms' optimization problem

- A firm re-optimizing at date t will choose the price P_t^* that maximizes the current market value of the profits generated while this price remains effective:

$$\underset{P_t^*}{Max}\mathbb{E}_t\left\{\sum_{k=0}^{+\infty}\theta^k Q_{t,t+k}\left[P_t^* Y_{t+k|t}-\Psi_{t+k}\left(Y_{t+k|t}\right)\right]\right\},$$

where

  - $Q_{t,t+k}\equiv\beta^k\left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+k}}$ is the stochastic discount factor for nominal payoffs between t and $t+k$,
  - $Y_{t+k|t}$ is output at $t+k$ for a firm that last reset its price at t,
  - $\Psi_t(.)$ is the nominal cost function at t,

subject to $Y_{t+k|t}=\left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon}C_{t+k}$ for $k\in\mathbb{N}$,

taking as given C_{t+k} and P_{t+k} for $k\in\mathbb{N}$.

FOC I

- Using $\partial Y_{t+k|t} / \partial P_t^* = -\varepsilon Y_{t+k|t} / P_t^*$, we get the following FOC:

$$\mathbb{E}_t \left\{ \sum_{k=0}^{+\infty} \theta^k Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \mathcal{M} \psi_{t+k|t} \right) \right\} = 0,$$

where $\psi_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t})$ denotes the nominal marginal cost at $t+k$ for a firm that last reset its price at t, and $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon - 1}$.

- Under flexible prices ($\theta = 0$), this FOC collapses to $P_t^* = \mathcal{M} \psi_{t|t}$, so that $\mathcal{M}$ is the "**desired markup**" (or frictionless markup).

- Dividing by P_{t-1}, we get

$$\mathbb{E}_t \left\{ \sum_{k=0}^{+\infty} \theta^k Q_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t-1}} - \mathcal{M} MC_{t+k|t} \Pi_{t-1,t+k} \right) \right\} = 0,$$

where $\Pi_{t-1,t+k} \equiv \frac{P_{t+k}}{P_{t-1}}$ and $MC_{t+k|t} \equiv \frac{\psi_{t+k|t}}{P_{t+k}}$ is the real marginal cost at $t+k$ for a firm whose price was last set at t.

FOC II

- Re-arranging the terms, we get

$$\mathbb{E}_t \left\{ \sum_{k=0}^{+\infty} \theta^k Q_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t-1}} - 1 \right) \right\} = \mathbb{E}_t \left\{ \sum_{k=0}^{+\infty} \theta^k Q_{t,t+k} Y_{t+k|t} \left(\mathcal{M} MC_{t+k|t} \Pi_{t-1,t+k} - 1 \right) \right\}.$$

- For small fluctuations (and in particular small fluctuations of Π_t around 1), we have $Q_{t,t+k} \approx \beta^k$, $Y_{t+k|t} \approx Y$,

$$\frac{P_t^*}{P_{t-1}} - 1 \approx \log \left(1 + \frac{P_t^*}{P_{t-1}} - 1 \right) = p_t^* - p_{t-1},$$

$$\mathcal{M} MC_{t+k|t} \Pi_{t-1,t+k} - 1 \approx \log \left(1 + \mathcal{M} MC_{t+k|t} \Pi_{t-1,t+k} - 1 \right) = \mu + mc_{t+k|t} + p_{t+k} - p_{t-1},$$

where $\mu \equiv \log \mathcal{M}$.

FOC III

- So, as a first-order approximation, we can rewrite the FOC as

$$\sum_{k=0}^{+\infty}(\beta\theta)^k Y(p_t^* - p_{t-1}) = \mathbb{E}_t \left\{\sum_{k=0}^{+\infty}(\beta\theta)^k Y(\mu + mc_{t+k|t} + p_{t+k} - p_{t-1})\right\}$$

and hence

$$p_t^* - p_{t-1} = (1-\beta\theta)\mathbb{E}_t \left\{\sum_{k=0}^{+\infty}(\beta\theta)^k \left(\mu + mc_{t+k|t} + p_{t+k} - p_{t-1}\right)\right\}.$$

- Adding p_{t-1}, we finally get

$$p_t^* = \mu + (1-\beta\theta)\mathbb{E}_t \left\{\sum_{k=0}^{+\infty}(\beta\theta)^k \left(mc_{t+k|t} + p_{t+k}\right)\right\}.$$

- Hence, firms resetting their prices choose a price equal to their **desired markup** over a **weighted average** of their current and expected future **nominal marginal costs**.

Market-clearing conditions

- Market clearing in the **goods markets** requires, for all i and t,

$$Y_t(i) = C_t(i).$$

- Therefore, $Y_t = C_t$, where $Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$.

- Market clearing in the **labor market** requires, for all t,

$$N_t = \int_0^1 N_t(i) di.$$

Aggregate production function

- Using the market-clearing conditions, the production function, and the demand schedule, one gets

$$N_t = \int_0^1 \left[\frac{Y_t(i)}{A_t} \right]^{\frac{1}{1-\alpha}} di = \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left[\frac{P_t(i)}{P_t} \right]^{\frac{-\varepsilon}{1-\alpha}} di,$$

and therefore the **aggregate production function**

$$y_t = (1-\alpha) n_t + a_t - d_t,$$

where $d_t \equiv (1-\alpha) \log \int_0^1 \left[\frac{P_t(i)}{P_t} \right]^{\frac{-\varepsilon}{1-\alpha}} di$ is a **measure of price dispersion** (and, hence, output dispersion) across firms.

Average real marginal cost

- For small fluctuations of Π_t around 1, d_t is of second order, so that the **aggregate production function** is approximated, at the first order, as

$$y_t = (1-\alpha)n_t + a_t$$

(the proof is relegated to the Appendix of Chapter 2).

- Noting mc_t the **average real marginal cost**, mpn_t the average marginal product of labor, and $\tau \in (0,1)$ a constant employment subsidy (financed by lump-sum taxes), and using $y_t = (1-\alpha)n_t + a_t$, we get

$$\begin{aligned} mc_t &= \log(1-\tau) + (w_t - p_t) - mpn_t \\ &= \log(1-\tau) + (w_t - p_t) - (a_t - \alpha n_t) - \log(1-\alpha) \\ &= \log(1-\tau) + (w_t - p_t) - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha). \end{aligned}$$

Real marginal cost

- For any firm-level variable z, let $z_{t+k|t}$ denote the value of z at $t+k$ for a firm that last reset its price at t.

- Using the demand schedule and the goods-market clearing condition, we get, at the first order, the **real marginal cost**

$$\begin{aligned} mc_{t+k|t} &= \log(1-\tau) + (w_{t+k} - p_{t+k}) - mpn_{t+k|t} \\ &= \log(1-\tau) + (w_{t+k} - p_{t+k}) - \frac{a_{t+k} - \alpha y_{t+k|t}}{1-\alpha} - \log(1-\alpha) \\ &= mc_{t+k} + \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \\ &= mc_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(p_t^* - p_{t+k}). \end{aligned}$$

- Under **constant returns to scale** ($\alpha = 0$), we have $mc_{t+k|t} = mc_{t+k}$: the **real marginal cost** is independent of the output level and, therefore, is **common across firms**.

Rewriting firms' FOC I

- Replacing $mc_{t+k|t}$ by the expression obtained, we rewrite firms' FOC as

$$p_t^* - p_{t-1} = (1-\beta\theta)\mathbb{E}_t\left\{\sum_{k=0}^{+\infty}(\beta\theta)^k\left[\Theta\left(\mu + mc_{t+k}\right) + p_{t+k} - p_{t-1}\right]\right\},$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$.

- Then, using $(1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k(p_{t+k} - p_{t-1}) = \sum_{k=0}^{+\infty}(\beta\theta)^k\pi_{t+k}$, we rewrite firms' FOC as

$$p_t^* - p_{t-1} = \mathbb{E}_t\left\{\sum_{k=0}^{+\infty}(\beta\theta)^k\left[(1-\beta\theta)\Theta\left(\mu + mc_{t+k}\right) + \pi_{t+k}\right]\right\},$$

and hence, in a recursive way, as

$$p_t^* - p_{t-1} = \left[(1-\beta\theta)\Theta\left(\mu + mc_t\right) + \pi_t\right] + \beta\theta\mathbb{E}_t\left\{p_{t+1}^* - p_t\right\}.$$

Rewriting firms' FOC II

- Using the aggregate price level dynamics equation

$$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right)$$

at dates t and $t+1$ to replace p_t^* and p_{t+1}^* in firms' FOC, and using $\pi_t \equiv p_t - p_{t-1}$, we then get

$$\pi_t = \beta \mathbb{E}_t \left\{\pi_{t+1}\right\} + \chi \left(\mu + mc_t\right),$$

where $\chi \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$.

Natural level of output

- Independently of the nature of price setting, the average real marginal cost can be rewritten, at the first order, as

$$\begin{aligned} mc_t &= \log(1-\tau) + (w_t - p_t) - mpn_t \\ &= \log(1-\tau) + (\sigma y_t + \varphi n_t) - (y_t - n_t) - \log(1-\alpha) \\ &= \log(1-\tau) + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha), \end{aligned}$$

  using the labor-consumption trade-off condition, the goods-market-clearing condition, and the (approximate) aggregate production function.

- Now, firms' FOC implies that, under flexible prices, $mc_t = -\mu$.

- Therefore, the **natural level of output**, defined as the equilibrium level of output under flexible prices and noted y_t^n, is such that

$$-\mu = \log(1-\tau) + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha).$$

Output gap and NKPC

- Therefore, the natural level of output is

$$y_t^n = \frac{1-\alpha}{\sigma(1-\alpha)+\varphi+\alpha}\left[\log\left(\frac{1-\alpha}{1-\tau}\right)+\frac{1+\varphi}{1-\alpha}a_t-\mu\right].$$

- The natural level of output does not depend on i_t, i.e. **monetary policy is neutral under flexible prices**.

- Subtracting the two equations on the previous slide, we get $mc_t+\mu = \left(\sigma+\frac{\varphi+\alpha}{1-\alpha}\right)\widetilde{y}_t$, where $\widetilde{y}_t \equiv y_t - y_t^n$ is called the **output gap**.

- Replacing $mc_t+\mu$ by this expression in firms' FOC, we eventually get the **New Keynesian Phillips curve** (NKPC)

$$\pi_t = \beta\mathbb{E}_t\left\{\pi_{t+1}\right\}+\kappa\widetilde{y}_t,$$

where $\kappa \equiv \chi\left(\sigma+\frac{\varphi+\alpha}{1-\alpha}\right)$.

Interpretation of the NKPC

- The NKPC is **forward-looking** because, when a firm resets its price, it knows that it will not be able to change this price for some (random) time.
- Therefore, the current inflation rate depends on
  - the current situation (term $\kappa \widetilde{y}_t$),
  - the expected future situation (term $\beta \mathbb{E}_t \{\pi_{t+1}\}$).
- The slope κ of the Phillips curve is decreasing in θ and β: the stickier the prices or the higher the discount factor, the less prices react to the current situation (relatively to the expected future situation).
- As the degree of price stickiness goes to zero ($\theta \to 0$), the NKPC slope goes to infinity ($\kappa \to +\infty$), and therefore the NKPC becomes $y_t = y_t^n$.

IS equation and natural rate of interest

- Using the goods-market-clearing condition and the definition of the output gap, one can rewrite the Euler equation as the **IS equation**

$$\widetilde{y}_t = \mathbb{E}_t\left\{\widetilde{y}_{t+1}\right\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left\{\pi_{t+1}\right\} - r_t^n\right),$$

where

$$\begin{aligned} r_t^n &\equiv r + \sigma \mathbb{E}_t\{\Delta y_{t+1}^n\} \\ &= r + \frac{\sigma(1+\varphi)}{\sigma(1-\alpha)+\varphi+\alpha}\mathbb{E}_t\{\Delta a_{t+1}\} \end{aligned}$$

is the **natural rate of interest** (unique equilibrium value of the ex ante short-term real interest rate $i_t - \mathbb{E}_t\left\{\pi_{t+1}\right\}$ consistent with the output level being constantly equal to its natural level).

Set of equilibrium conditions

- Given $(a_t, i_t)_{t\in\mathbb{N}}$, $(\widetilde{y}_t, \pi_t)_{t\in\mathbb{N}}$ is determined by the
  - **IS equation** $\widetilde{y}_t = \mathbb{E}_t\left\{\widetilde{y}_{t+1}\right\} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left\{\pi_{t+1}\right\} - r_t^n\right)$ for $t \in \mathbb{N}$,
  - **NKPC** $\pi_t = \beta\mathbb{E}_t\left\{\pi_{t+1}\right\} + \kappa\widetilde{y}_t$ for $t \in \mathbb{N}$,

  which implies that **monetary policy is not neutral** (unless $\theta = 0$).

- Given $(a_t, i_t, \widetilde{y}_t, \pi_t)_{t\in\mathbb{N}}$, $(y_t, c_t, n_t, w_t - p_t)_{t\in\mathbb{N}}$ is determined by the
  - definition of the output gap $\widetilde{y}_t \equiv y_t - y_t^n$ for $t \in \mathbb{N}$,
  - goods-market-clearing condition $c_t = y_t$ for $t \in \mathbb{N}$,
  - aggregate production function $y_t = (1-\alpha)n_t + a_t$ for $t \in \mathbb{N}$,
  - labor-consumption trade-off condition $w_t - p_t = \sigma c_t + \varphi n_t$ for $t \in \mathbb{N}$.

Role of expectations I

- Provided that $\lim_{k\rightarrow+\infty} \mathbb{E}_t\left\{\widetilde{y}_{t+k}\right\}=0$, iterating the IS equation forward yields

$$\widetilde{y}_t=-\frac{1}{\sigma}\mathbb{E}_t\left\{\sum_{k=0}^{+\infty}\left(r_{t+k}-r_{t+k}^n\right)\right\},$$

  where $r_t \equiv i_t-\mathbb{E}_t\left\{\pi_{t+1}\right\}$ is the ex ante **short-term** real interest rate.

- Using a no-arbitrage condition, we can interpret $\mathbb{E}_t\left\{\sum_{k=0}^{+\infty} r_{t+k}\right\}$ as the ex ante **long-term** real interest rate.

- Therefore, the current output gap depends on the **private sector's expectations of the future path of the short-term interest rate** (through the long-term interest rate).

Role of expectations II

- Provided that $\lim_{k \to +\infty} \mathbb{E}_t \{\pi_{t+k}\} = 0$, iterating the NKPC forward yields

$$\pi_t = \kappa \mathbb{E}_t \left\{ \sum_{k=0}^{+\infty} \beta^k \widetilde{y}_{t+k} \right\}.$$

- Therefore, the current inflation rate also depends on the **private sector's expectations of the future path of the short-term interest rate**.

- So, monetary policy affects the economy not only through changes in the **current short-term interest rate**, but also through changes in the **private sector's current expectations** of the future path of the short-term interest rate.

Role of expectations III

- As an **illustration**, assume for simplicity that the central bank controls directly r_t and follows the rule

$$r_t = \gamma r_{t-1} + \xi_t,$$

where $\gamma \in (0,1)$ and ξ_t is an i.i.d. monetary-policy shock.

- We then get $\mathbb{E}_t \left\{ \sum_{k=0}^{+\infty} r_{t+k} \right\} = r_t / (1-\gamma)$ and

$$\widetilde{y}_t = \frac{-r_t}{\sigma(1-\gamma)} + \text{exogenous terms},$$

$$\pi_t = \frac{-\kappa r_t}{\sigma(1-\gamma)(1-\beta\gamma)} + \text{exogenous terms}.$$

- So, the **more persistent** the short-term interest rate (i.e. the closer γ to 1), the **larger** the effect of the monetary-policy shock ξ_t on the current long-term interest rate, the current output gap, and the current inflation rate.

In Woodford's (2003) words I

"Successful monetary policy is not so much a matter of effective control of overnight interest rates as it is of shaping market expectations of the way in which interest rates, inflation, and income are likely to evolve over the coming year and later. (...) [O]ptimizing models imply that private sector behavior should be forward-looking; hence expectations about future market conditions should be important determinants of current behavior. It follows that, insofar as it is possible for the central bank to affect expectations, this should be an important tool of stabilization policy. (...) Not only do expectations about policy matter, but, at least under current conditions, very little else matters.

In Woodford's (2003) words II

[T]he current level of the overnight interest rates as such is of negligible importance for economic decisionmaking. The effectiveness of changes in central-bank targets for overnight rates in affecting spending decisions (and hence ultimately pricing and employment decisions) is wholly dependent upon the impact of such actions upon other financial-market prices, such as long-term interest rates, equity prices, and exchange rates. These are plausibly linked, through arbitrage relations, to the short-term interest rates most directly affected by central-bank actions. But it is the expected future path of short-terms rates over coming months and even years that should matter for the determination of these other asset prices, rather than the current level of short-term rates by itself. Thus the ability of central banks to influence expenditure, and hence pricing, decisions is critically dependent upon their ability to influence market expectations regarding the future path of overnight interest rates, and not merely their current level."

In Bernanke's (2004b) words I

"Informal discussions of monetary policy sometimes refer to the Fed as 'setting interest rates.' In fact, the FOMC does not set interest rates in general; rather, the Committee 'sets' one specific interest rate, the federal funds rate. The federal funds rate, the interest rate at which commercial banks borrow and lend to each other on a short-term basis (usually overnight) is not important in itself. Only a tiny fraction of aggregate borrowing and lending is done at that rate. From a macroeconomic perspective, longer-term interest rates–such as home mortgage rates, corporate bond rates, and the rates on Treasury notes and bonds–are far more significant than the funds rate, because those rates are the most relevant to the spending and investment decisions made by households and businesses. These longer-term rates are determined not by the Fed but by participants in deep and sophisticated global financial markets.

In Bernanke's (2004b) words II

Although the FOMC cannot directly determine long-term interest rates, it can exert significant influence over those rates through its control of current and future values of the federal funds rate. The crucial link between the federal funds rate and longer-term interest rates is the formation of private-sector expectations about future monetary policy actions. Loosely speaking, long-term interest rates embody the expectations of financial-market participants about the likely future path of short-term rates, which in turn are closely tied to expectations about the federal funds rate. Thus, to influence long-term interest rates, such as thirty-year mortgage rates or the yields on corporate bonds, the FOMC must influence private-sector expectations about future values of the federal funds rate. The Committee can do this by its communication policies, by establishing certain patterns of behavior, or both."